

# Dynamic Modeling of Liquid Crystal Display Cells using a Constant Charge Approach

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**Abstract--** We present a new formulation for the dynamic modelling of twisted nematic liquid crystal display cells based on a constant charge assumption instead of the commonly used fixed voltage model. This formulation is based on a tensor representation of the director field, which is solved using a combination of finite element and finite difference methods. It allows the accurate modeling of liquid crystal pixel cells in TFT driven active matrix displays, extensively used in flat panel computer and television displays.

**Index Terms—**Liquid Crystals, Modeling, Finite Element Methods

## I. INTRODUCTION

The accurate modelling of the dynamic behaviour of liquid crystal materials has become increasingly important as liquid crystal displays (LCDs) become more sophisticated and require a more careful and efficient design. Characteristics like switching time, flicker, and viewing angle are important effects that need to be properly and accurately modelled in the design process. High resolution LCDs with active matrix addressing present another set of challenges to the designer. These devices drive the pixel cells using active components, normally a thin film transistor (TFT) located in the same pixel area to select and switch the individual pixels in the matrix as shown schematically in Figs. 1 and 2. Twisted nematic (TN) TFT-driven devices operate by connecting the signal voltage to the pixel electrode via a TFT for a very small fraction of the time, typically 15-30  $\mu\text{s}$  in each frame (of about 16 ms). The TFTs are switched on sequentially row by row, by applying a short pulse to the gates while the data for a particular row is applied via the column electrodes to the source of the transistors. The pixel electrode will have a voltage of approximately the form shown in Fig. 1, alternating in polarity on successive frames, with a period of around 33ms. The polarity changes so as to prevent a net D.C. voltage from being applied to the pixel, which over a large number of frames can cause degradation of the liquid crystal. A charge is deposited in the pixel electrode during this connection time leaving the voltage floating while

all of the other rows in the display are addressed successively. As the liquid crystal switches with the applied electric field, the permittivity, dependent on the orientation of the molecules, changes and since the charge remains constant in the isolated electrode, the pixel voltage drifts from the set value as shown schematically in Fig. 2 (top right). This unwanted effect produces deterioration of the response time and image flicker as the voltage is reset in the next frame. External storage capacitors are used to reduce this effect (as shown with dotted lines in Fig. 2). TN-TFT displays cannot be properly modelled using previous models based on the assumption of a fixed, known voltage applied all the time to the pixel electrodes [1-3]. The method presented here uses an approach to the calculation of the director and potential distribution similar to previous methods but starts by calculating the pixel voltages from the charges deposited on the electrodes. Flicker and the effect of external storage capacitors are easily modelled. In addition, as part of the calculation process, the varying capacitances (self and mutual) of all electrodes are calculated

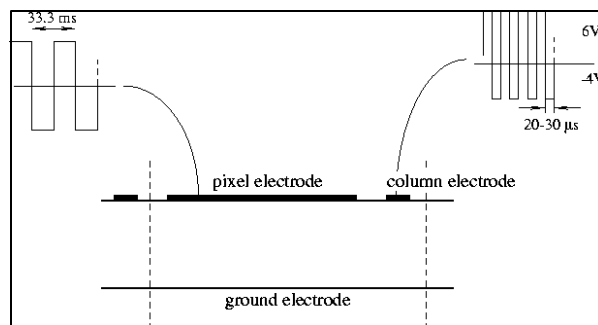


Fig.1 Cross-section of pixel cell and pixel and column signals.

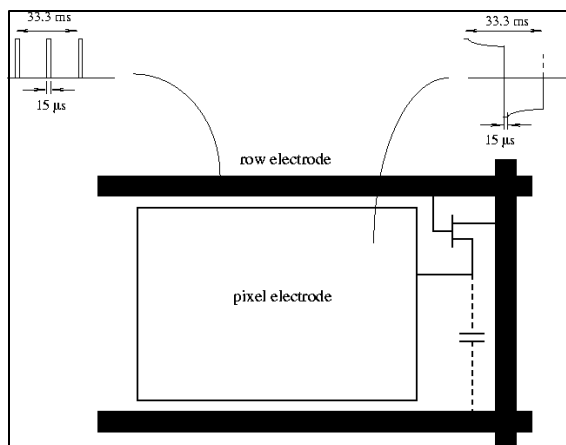


Fig. 2 Diagram of a display pixel cell and voltages.

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as functions of time. These are useful in the design of display back planes.

## II. DESCRIPTION OF THE METHOD

The macroscopic behaviour of liquid crystal materials is commonly represented by the director  $\mathbf{n}$ , a unit vector that indicates the local average orientation of the molecules. However, since the elongated molecules are symmetric, a vectorial representation can be inappropriate if it breaks this head-tail symmetry, introducing spurious numerical artifacts.

In common with our previous “fixed voltage” model [1-3], this new formulation uses a tensor representation of the liquid crystal local orientation field, based on the symmetric dyadic  $\mathbf{nn}$ , and is based on a dynamic solution for the Oseen-Frank formulation describing the balance of elastic and electric energies in the device [4]. Using an approximate simple model that describes the elastic properties of the material using only one elastic constant, this formulation becomes:

$$n_n \left[ \mathbf{g} \frac{\partial}{\partial t} (n_n n_m) - K \nabla^2 (n_n n_m) - \mathbf{e}_0 \Delta \mathbf{e} E_n E_m \right] = 0 \quad (1)$$

where repeated indices imply summation,  $\mathbf{m} \mathbf{n} = x, y$  or  $z$ ,  $n_n$  is the  $\mathbf{n}$ -component of the director  $\mathbf{n}$ ,  $\mathbf{De}$  describes the dielectric anisotropy of the LC material,  $E$  is the applied electric field,  $\mathbf{g}$  is the rotational viscosity and  $K$  is the elastic constant that

describes the elastic deformation.

Combining equation (1) with the Laplace equation to calculate the electric field and using the time-varying voltages at the electrodes as input, this set of equations is solved using a Crank-Nicolson time stepping procedure. Due to the highly non-linear relation between director and potential distribution in the cell, the calculation of these quantities within each time step is performed iteratively, first assuming a fixed director distribution and solving for the electric potential using finite elements, followed by solving for the director distribution using finite differences, now assuming a fixed electric potential. This process is repeated, within the same time step, until consistency between potential and director distributions is achieved. Fig. 3 shows schematically the flow diagram of the calculation procedure.

The signal voltage is applied to the pixel electrodes through the TFTs only during the ON time of the transistors, a small fraction of the frame time and during the rest of the time the voltage on these electrodes is left floating but the charge remains constant. At the last time step with a fixed voltage, the charge density over the electrodes is calculated from the potential distribution. The total charge on the pixel electrodes will then be considered constant for the time the TFT is not conducting. The corresponding capacitance matrix between the electrodes is calculated next and then, the resultant new voltages at the electrodes can be determined. The process can then continue in the same form as for the ‘fixed voltage’ model [5,6]. Further iterations are needed however, before completing the time step in order to ensure that the charge is indeed kept constant on the pixel electrode. Normally no more than two iterations are needed.

Practical display devices normally use external capacitors connected to each pixel cell in order to palliate the effect of the voltage drift. The method described here allows in a straightforward manner the inclusion of these capacitors in the model by simply adding them to the corresponding elements of the instantaneous capacitance matrix.

### 1) Calculation of the potential

This is performed at each time step, alternated with the director calculation and repeated until consistence is achieved, using a finite element solution of the Laplace equation. Directors are calculated using finite differences and both calculations are performed with the same mesh of nodes. In many practical cases, it is adequate to reduce the problem to two dimensions by considering only a transverse cross-section of the cell and assuming invariance in the other coordinate. Triangular first order finite elements are used for the potential in this case. In the full 3D approach, a mesh of regular first order brick elements is used in order to be able to share the mesh nodes with the finite difference calculation of the director. In both cases, the permittivity tensor is described in each element by:

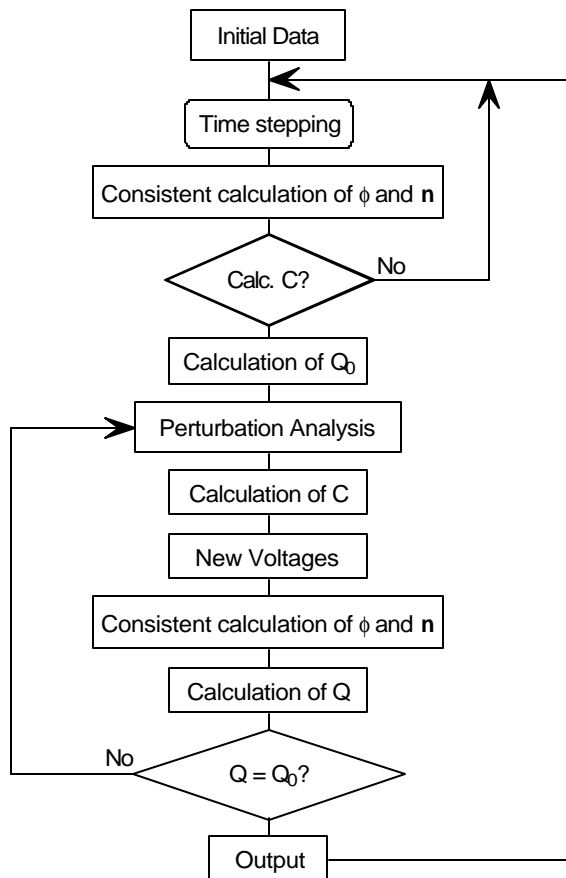


Fig. 3 Flow chart describing the calculation procedure

$$\bar{\mathbf{e}} = \Delta \mathbf{e} \hat{\mathbf{n}} + \mathbf{e}_{\perp} \bar{\mathbf{I}} \quad (2)$$

where  $\Delta \mathbf{e} = \mathbf{e}_{\parallel} - \mathbf{e}_{\perp}$  and  $\mathbf{e}_{\parallel}$ ,  $\mathbf{e}_{\perp}$  are the parallel and perpendicular permittivities (with respect to the optical axis). The director values are known at nodes of the mesh and they are represented over the entire element using the same shape functions as the potential when they are used in (2). This will lead in the 3D case, to the calculation of local integrals of the form:

$$J_{ijkl} = \frac{V}{8} \int \mathbf{y}_i \mathbf{y}_j \frac{\partial \mathbf{y}_k}{\partial u} \frac{\partial \mathbf{y}_l}{\partial v} dudvdw \quad (3)$$

for example, where  $\mathbf{y}_i$  are first order shape functions defined over the brick elements (8 nodes) and  $u, v, w$  are the normalised element co-ordinates. The indices  $i, j, k, l$  can vary from 1 to 8, giving a total of 4096 different combinations. However, there will be only a few different results. The identification of the correct value of these integrals from the index combination can be very time consuming during the matrix assembly process. In this case this is done by evaluating a set of logical functions that represent the symmetries in the brick element with respect to the evaluation of (3). Representing each index as a sequence of three binary digits ABC, any combination  $ijkl$  corresponds to a twelve digit binary number  $A_n B_n C_n$ ,  $n=1,2,3,4$ . With this, the value of  $J_{ijkl}$  can be written as:

$$J_{ijkl} = \frac{\pm V}{2\Delta x^2 8^4} \left\{ \begin{array}{c} S \\ D \end{array} \right\} \left\{ \begin{array}{c} K_1 \\ L_1 \\ M_1 \end{array} \right\} \left\{ \begin{array}{c} K_2 \\ L_2 \\ M_2 \end{array} \right\} \quad (4)$$

where  $S, D, K_1, L_1, M_1, K_2, L_2, M_2$  are logical functions of  $A_n$  and  $B_n$ . For example:

$$\begin{aligned} K_1 &= \bar{B}_1 \bar{B}_2 \bar{B}_3 \bar{B}_4 + B_1 B_2 B_3 B_4 \\ L_1 &= B_1 \oplus B_2 \oplus B_3 \oplus B_4 \\ M_1 &= \bar{B}_1 \bar{B}_2 B_3 B_4 + \bar{B}_1 B_2 \bar{B}_3 B_4 + \bar{B}_1 B_2 B_3 \bar{B}_4 \\ &\quad + B_1 B_2 \bar{B}_3 \bar{B}_4 + B_1 \bar{B}_2 B_3 \bar{B}_4 + B_1 \bar{B}_2 \bar{B}_3 B_4 \end{aligned} \quad (5)$$

## 2) Calculation of the charge and capacitance

The total charge over each electrode is calculated by numerical integration of the charge density obtained from the evaluation of the normal derivative of the potential at the electrodes, including the appropriate singular behaviour at the electrode edges. This is particularly important in the evaluation of the charge on drive (row and column) electrodes since these are much narrower than the pixel electrodes and the edge effects will have a marked effect on the total charge [5,6]. The normal derivatives are calculated using up to 5 points of the finite difference mesh in order to obtain adequate accuracy. A curve that fits the charge density values in the vicinity of the edges and the known singular behaviour at the edges is then calculated and used to evaluate the total charge on the

electrodes.

For the corresponding director distribution (instantaneous switching state of the device), the generalised, instantaneous capacitance matrix between all electrodes in the cell can be calculated and this allows the determination of the electrode voltages. Iterations are again necessary to ensure consistency between the calculated values. The capacitance matrix is calculated using a perturbational approach, which is accurate since in this case the problem is linear. Applying a fictitious test voltage to the electrodes in the cell one by one with a frozen director field, the change in the charge is calculated in all electrodes, giving the corresponding terms of the capacitance matrix. That is, using the relation  $dQ = C dV$ , for each  $dV_j$  applied at electrode  $j$  the charge increment  $dQ_i$  in electrode  $i$  will allow to calculate the value of  $C_{ij}$ . However, the calculation needs to be done applying a voltage to each electrode only once and in sequence to obtain the matrix  $C$  row by row. The symmetry of the matrix can be used to avoid inaccurate calculations involving finding the charge in drive electrodes. The use of external storage capacitors can then be simply simulated by just adding the corresponding capacitance to the self capacitance of the pixel electrodes.

## III. RESULTS

Fig.4 shows the drifting of the pixel voltage as the cell is switched, as happens in a TFT driven cell, compared to the case where the voltage is kept constant. The rapid fluctuations are due to the coupling effect between the pixel and drive electrodes through the fluctuating electric field (mutual capacitance between pixel and drive electrodes). A comparison is clear with the case of a 'constant voltage' model, where the pixel electrode always has an externally applied voltage. In this test example the drive voltage varies between  $\pm 5V$  with a period of 0.66 ms while the pixel voltage is kept at  $\pm 4V$ , enough to produce full switching of the liquid crystal, with a frame duration of 16.6667ms. This voltage is applied to the pixel electrode only during the first 20 $\mu$ s of each

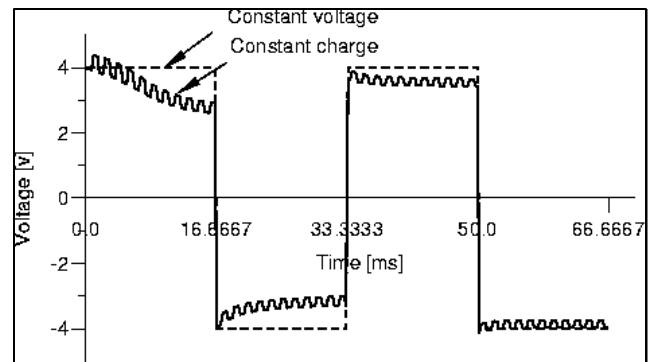


Fig. 4. Variation of pixel voltage with time

frame. The drifting of the pixel voltage is quite noticeable, particularly at the beginning of the switching process in the first frame. In the successive frames the drift becomes smaller as the liquid crystal reaches its switched state.

The effect of the voltage drift can also be seen in Figs. 5 and 6, on the pixel self-capacitance and the cell optical transmittance. Fig. 6 also shows the effect of external capacitors of different size. It can be observed that the voltage drift causes a slowing down of the time response of the cell, and that this is improved by the introduction of an external storage capacitor. However, the best value for this capacitor is not as high as possible (to have a constant voltage on the pixel electrodes) but some intermediate value instead, – in this case about 10 nF.

The voltage drift will also produce flicker in the cell optical transmittance. Fig. 7 shows the effect of the drifting pixel voltages on the optical transmittance of the cell along several frames while switching to an intermediate grey level (30% transmittance).

#### IV. CONCLUSIONS

The method described here provides a much more accurate simulation in two or three dimensions of the dynamic behaviour of real TFT-driven pixel cells in active matrix liquid crystal displays than the commonly used procedures. Flicker effects due to drifting voltages are easily modelled as well as the effect of external storage capacitors. The results from the modelling also provide accurate values of capacitance (self and mutual) to aid in the design of back planes.

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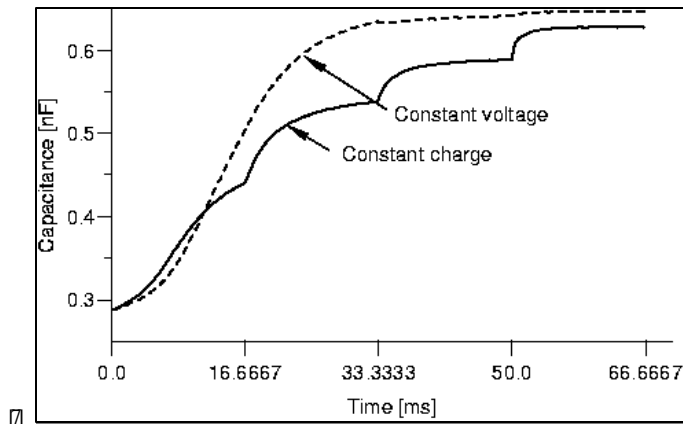


Fig. 5 Pixel self-capacitance varying with time.

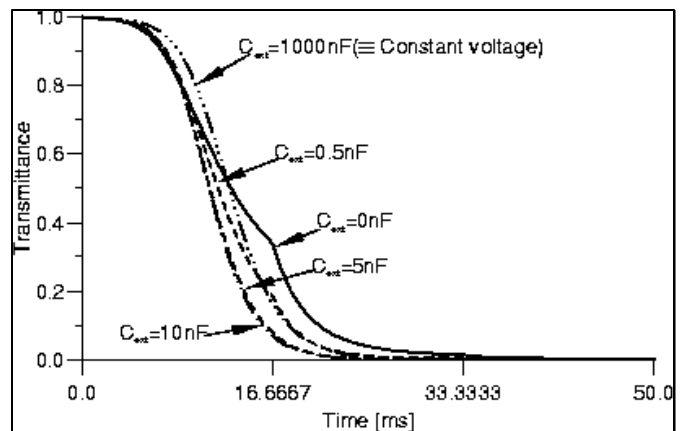


Fig. 6 Optical transmittance time variation with different external storage capacitance.

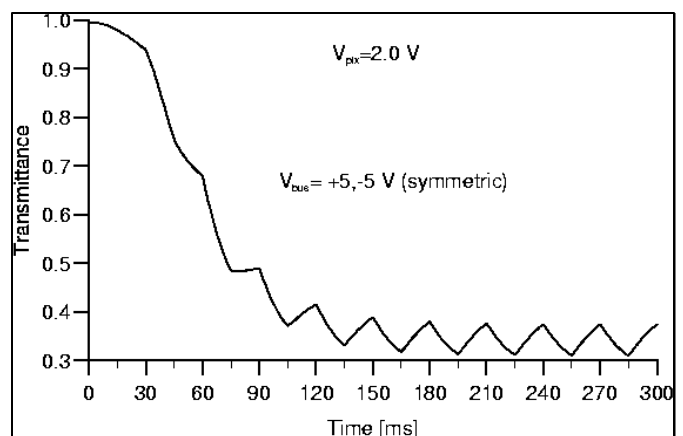


Fig. 7 Flicker effect of voltage drift in a cell switching to a grey level.

## REMARKS TO THE REVIEWERS

A more detailed comparison with experimentally observed results was not possible due to the lack of data for this comparison. Flicker effects have been reported mostly in a qualitative manner and our results demonstrate their presence and provide tools for the design of countermeasures. Comparison is made with the results of the 'constant voltage' model, an already established model for the approximate simulation of these devices.

An extensive explanation of the background of the problem, - on how the electronics associated with the operation of the display produce the effect studied, has been included in the introduction to this extended version.

Some more explanation about the origin of equation (1) is also given, as a balance of electric and elastic energies. Full derivation is not possible and the reader is referred to its source (our ref. [4]). That paper was dedicated entirely to its derivation.