Simulation of Refraction, Retardation and Transmission in Liquid Crystal Displays with Slow Lateral Variations

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Abstract

Slow lateral variations in the liquid crystal properties distort the shape of an incident wavefront. The lateral variation in the phase, obtained with the extended Jones calculus, is used to determine refraction effects. Refraction depends on the polarization state of the light and the resulting transmission through the liquid crystal may be very different from what is obtained with the Jones calculus.

Introduction

In several recently developed liquid crystal display (LCD) technologies, the director orientation varies as a function of the lateral position on the glass substrate. The resulting inhomogeneous dielectric tensor distorts the wavefront of an incident plane or spherical wave and this may influence the transmission characteristics of the LCD considerably. The aim of this paper is to investigate to what extent small lateral variations in the director orientation will influence the transmission in realistic liquid crystal devices.

An accurate treatment of this problem depends largely on the length scale of the lateral inhomogeneity. If the scale of the variation in index of refraction is sufficiently large compared to the wavelength of the light, it is acceptable to determine the transmission at a given location with the Jones calculus, using local values for the director orientation. When on the other hand the scale of a periodic lateral variation is much smaller than the wavelength, the wavefront will basically react to the average of the permittivity tensor [1]. In these two extreme cases, incoming plane waves are transformed into outgoing plane waves with the same *k*-vector. In the intermediate case, where the scale of the variations is similar to the wavelength of the light source, light is scattered and diffraction effects play an important role.

In this paper we present a method to study the effect of lateral variations on a scale which is relatively large compared to the wavelength, which makes it possible to consider diffraction effects as a perturbation on the Jones calculus.

Diffraction and Refraction

Figure 1 shows the setup of a spherical wave incident on a thin layer with lateral variations in the index of refraction n(x). With this example we will illustrate the approximation of 'slow lateral variation' of the index of refraction with respect to *x*.

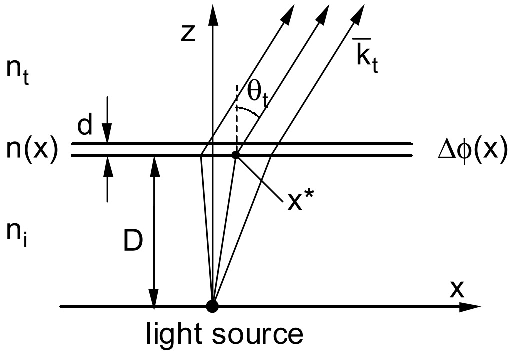

Fig. 1 Setup for diffraction and refraction.

According to the Fresnel theory for diffraction [2], the complex amplitude is modulated as a function of the transmission direction:

$$U(k_{xt}) = \int_{-\infty}^{\infty} \exp\left[jk\, n_i \sqrt{D^2 + x^2} + j\Delta\phi - jk_{xt} x\right] dx \tag{1}$$

with k_{xt} the *x* component of the *k*-vector of the transmitted light:

$$k_{xt} = \frac{2\pi}{\lambda} n_t \sin(\theta_t)$$

The first term in the exponent gives the phase delay before the layer is reached; $\Delta\phi$ is the phase delay in the layer. The above diffraction formula suggests that the entire layer contributes to the transmitted light for a particular k_{xt}. However, in the case that the variation of $\Delta\phi$ along the *x* axis is small, the main contribution in the integral is from the *x*-interval where the phase is stationary. Setting the derivative of the phase equal to zero yields:

$$k_{xt} = k\, n_i \frac{x}{\sqrt{D^2 + x^2}} + \frac{d\Delta\phi}{dx} \tag{2}$$

For a given k_{xt}, this equation has a unique solution x^*

if the lateral variations of $\Delta\phi$ are slow:

$$\left|\frac{d^2\Delta\phi}{dx^2}\right| << k\,n_i\,\frac{D^2}{\left(D^2+x^2\right)^{3/2}} \tag{3}$$

The complex amplitude U in equation 1 can then be calculated by keeping only the quadratic term in the Taylor series expansion of the phase:

$$\left|U(k_{xt})\right| = \sqrt{\frac{\pi}{k\,n_i\,\frac{D^2}{\left(D^2+x^{*2}\right)^{3/2}}+\frac{d^2\Delta\phi}{dx^2}(x^*)}} \tag{4}$$

Based on the Fresnel diffraction integral 1, we have thus found that under certain conditions 3, the inhomogeneous layer refracts the light according to equation 2. The approximation is better if the lateral variation in ϕ is smaller, if the wavelength is shorter or if the light source is closer to the layer.

A refraction formula equivalent to equation 2 can be found by setting the optical path lengths of the two waves in figure 2 equal:

$$k_{xt} = k_{xi} + \frac{d\Delta\phi}{dx} \tag{5}$$

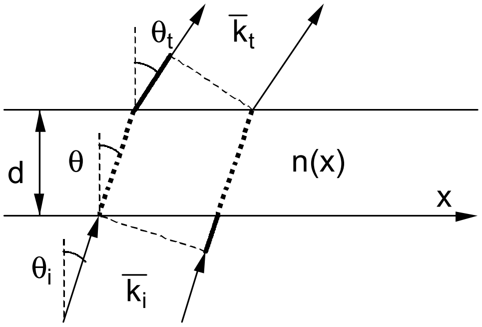

Fig. 2 Transmission of two rays through an inhomogeneous layer with slow lateral variations.

Figure 3 shows the calculation results with the Fresnel diffraction formula for two values of D: 1 m and 500 μm. The other parameters are: λ=0.5 μm; $\Delta\phi = \frac{\pi}{2}\cos\left(\frac{2\pi x}{L}\right)$ and the period of the variation L=50 μm. When the light source is far, diffraction peaks with spacing $\Delta\theta=\lambda/L$ are visible. When the light source is close enough, refraction peaks for individual lines are observed with spacing $\Delta\theta=L/D$.

At this point it is worth mentioning that for non-absorbing layers, the different kinds of diffraction and refraction patterns spread out or disappear completely if the lateral dimensions of the light source are large enough.

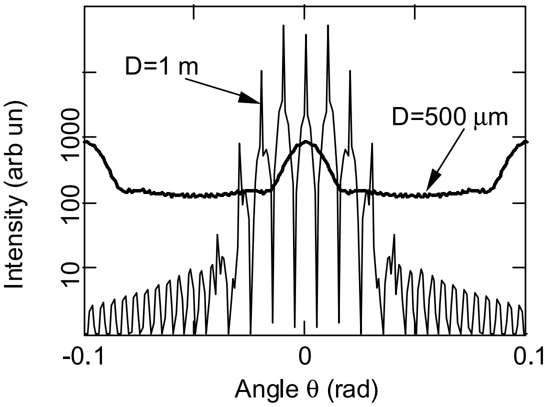

Fig. 3 Transmitted intensity $|U|^2$ as a function of emission angle θ_t for a spherical wave through a grid with oscillating *n(x)*, for D= 500 μm and 1 m.

Refraction and Retardation in a LCD

In this section we apply the theory of refraction at layers with slow lateral variations to the case of liquid crystal layers. As the liquid crystal material is anisotropic, incoming light waves are decomposed into two linearly polarized waves, which will generally be refracted in a different way. This difference in refraction determines the polarization of the light arriving at the analyser and thus the transmission of the liquid crystal device. The issues of illumination, refraction and retardation in liquid crystals are discussed.

1. Illumination

In most monitor liquid crystal display applications the emission from the illuminator is rather incoherent because it passes through a diffusor before reaching the liquid crystal material. The light from the diffusor is composed of a large number of mutually incoherent waves. Each of these waves is related to a specific region of the diffusor and can roughly be considered as a coherent spherical wave.

For each spherical wave, the transmission calculation is based on coherent light, thus the complex amplitudes must be added, and phase retardations between different polarisations have to be taken into account. Because different spherical waves are incoherent, the total transmission through the liquid crystal is found by adding the transmitted energies for a large number of spherical waves. In the following, we will only discuss coherent contributions from an elementary spherical wave.

2. Refraction

As an example we consider a liquid crystal device in which the molecules are oriented parallel to the *xz*-plane, with a tilt angle depending on the *x* and *z* coordiates (figure 4).

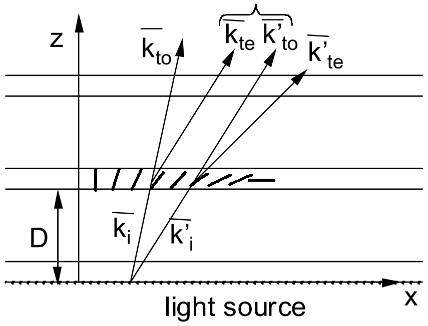

Fig. 4. Transmission of *o* and *e* waves through a liquid crystal; addition of waves with the same *k*-vector

This is approximately the case for in-plane switching devices near the edge of the electrode, when a voltage is applied [3,4]. In the following it is assumed that the light is incident in the same *xz*-plane. With the extended Jones calculus [5], we can find the phase difference $\Delta\phi(x_0,\theta_i)$ between top and bottom of the LC at x_0 for a wave with incident angle θ_i, under the assumption that the refractive index tensor does not depend on the *x* coordinate: $n(x,z) = n(x^*,z)$. This phase difference depends on the polarization of the incident light.

Within the Jones calculus the transmitted wave has the same *k* vector as the incident wave, because the phase differences do not depend on *x*. When the calculation procedure is repeated for different values of x^*, the dependence on the *x*-coordinate is introduced and the ordinary (*o*) and extraordinary (*e*) incident waves are refracted according to equation 5. Note that the derivative of the phase difference is normally not the same for *o* and *e* waves, so the refracted *o* and *e* waves are not parallel anymore, as illustrated in figure 4.

Figure 5 shows the variation of the k_x vector for *o* and *e* waves as a function of *x* for a liquid crystal device with the following parameters: $n_g=n_o=1.5$; $n_e=1.55$; director tilt $\theta(x)=\pi\, x/L$; $D=500$ µm; d=5 µm; $\lambda=0.5$ µm; $L=25$ or 50 µm. In this structure the tilt varies periodically with *x*, with period *L*. Clearly, a shorter period, causes stronger oscillations in the k_x vector of the *e*-wave.

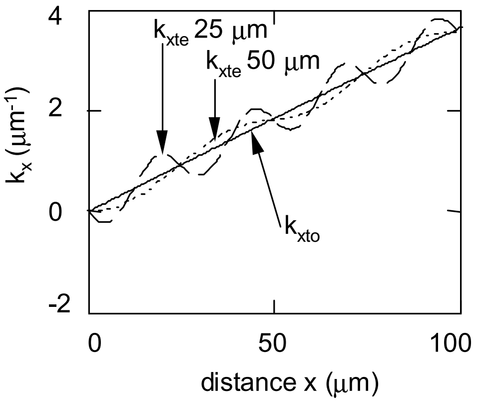

Fig. 5. Variation of the k_x-vector for *e* and *o* waves, for the example described in the text (L=25 or 50 µm).

3. Phase Retardation

In the Jones calculus, the transmitted *o* and *e* waves have the same *k*-vector and the phase retardation between them is simply given by $R_J = \Delta\phi_e - \Delta\phi_o$. After passing a liquid crystal layer with lateral variations, the transmitted *o* and *e* waves are refracted differently. For the calculation of the transmission through the second polarizer, it is important to add the complex amplitudes of the *o* and *e* contributions, which have the same wave vector *k*.

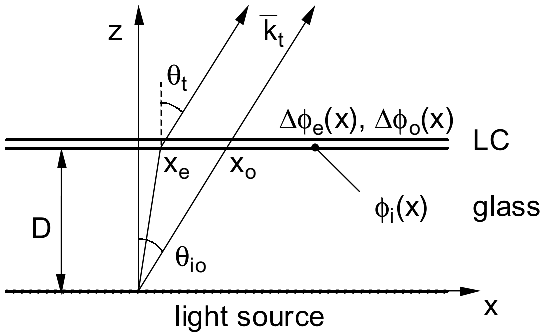

Fig. 6. Different paths for *o* and *e* waves, leading to a modified phase retardation.

The *o* and *e* contributions with the same k_{xt} are found at slightly different locations x_e and x_o, such that $k_{xte}(x_e) = k_{xto}(x_o)$. For a device with the same parameters as above, the distance x_e-x_o as a function of x_e is shown in figure 5. In the case of L=25 µm, the condition for small lateral variations 3 is not fulfilled and sometimes three *e*-waves with the same k_{xt} vector have to be added.

When the lateral distance x_e-x_o is taken into account, the phase retardation between the *e* and the *o* wave with the same k_{xt} vector is calculated as follows:

$$R = \phi_i(x_e) - \phi_i(x_o) + \Delta\phi_e(x_e) - \Delta\phi_o(x_o) - (x_e - x_o)\cdot k_{xte}(x_e) \qquad (6)$$

with the phase of the incoming wave given by:

$$\phi_i(x) = k\, n_g \sqrt{D^2 + x^2} \qquad (7)$$

This equation provides a first order correction factor on the Jones calculus, related with lateral variations in the liquid crystal. When the distance between x_e and x_o becomes zero it reduces to the Jones formula for retardation.

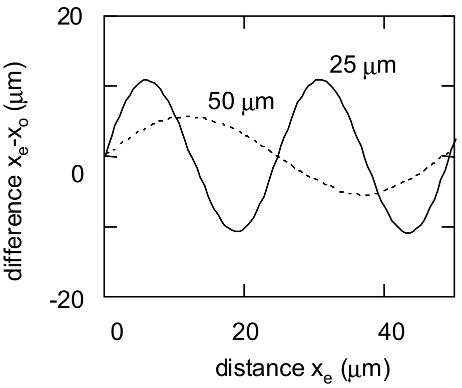

Figure 7. Distance x_e-x_o between *o* and *e* waves with the same wave vector (*L*=25 or 50 μm).

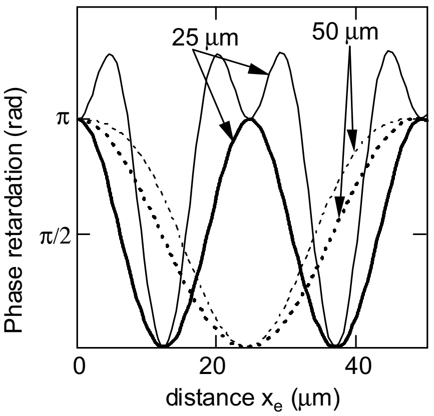

Figure 8. Calculated phase retardation with Eq. 6 (thin); with Jones calculus R_J (thick), *L*=25 or 50 μm.

4. Modulation of Amplitudes

In order to determine the transmission through the analyser, it is necessary to know the amplitudes of the *o* and *e* waves and the phase retardation between them. The amplitude of the transmitted light will be modulated according to equation 4. In the example of figure 6, it means that the transmitted *e* wave may be stronger or weaker than the *o* wave, even if the two incident beams have the same intensity.

In general, one can expect that lateral variations in the index of refraction will tend to modulate amplitudes and phase retardations of the different polarizations that pass through the liquid crystal layer. As a result, it will be more difficult to obtain a good bright or dark state, because the transmitted light will normally not be linearly polarized.

Conclusions

The presented simulation method for liquid crystal layers with lateral variations uses the phase information obtained from the Jones calculus, to determine refraction, retardation and amplitude modulation of light transmission. The corrections to the Jones calculus become more important when the length scale of the lateral variation becomes smaller, and when the distance between the light source and the LCD increases. Our description based on refraction is valuable to determine first order corrections to the Jones calculus, but when the deviations are too large, diffraction effects must be considered.

The simulations indicate that lateral variations in the liquid crystal have the tendency to mix up the polarization state of the incoming light, making it more difficult to obtain a good dark or bright state. Probably this qualitative conclusion will also hold when diffraction effects determine the light transmission.

Acknowledgements

This research work is sponsored by the MonLCD project of the European Commission (G5RD-CT 1999-00115); K. Neyts is with the FWO; K. Vermeirsch is with the IWT.

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